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Stopping power of nonmonochromatic heavy-ion clusters with two-ion correlation effects

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The stopping power of an ensemble of a large number of fast heavy ions, moving in a plasma, with a distribution function which has small spreads both in the physical and in the velocity spaces, is computed on the basis of a classical dielectric theory, retaining two-ion correlation effects. Averaging procedures in the configuration space are used to determine the actual friction force acting on the whole system and to evaluate the vicinage effects when the ion beam is not strictly monochromatic, as it was considered previously. Approximate analytical scalings with the physical parameters of the considered system of the *average vicinage function* and of the *decorrelation time* are given.

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In the last two decades the investigation of the interaction of energetic ions with matter under the plasma state has received particular attention due to its close concern with the implementation of thermonuclear reactors where fusion reactions should be realized at a sufficiently high rate to gain net energy [1]. Recently, it has also been proposed to use cluster ion beams as highly massive drivers for inertial confinement fusion (ICF), due to the lower beam current, weaker requirement on beam focusing, and smaller range achievable with this technique [2].

One aspect of particular interest of the slowing-down process of a molecular cluster in a plasma is that, during the process of Coulomb explosion [3] inside the medium, the charged debris are situated at a small distance (a few Å) from one another and their motion is highly correlated. Then the stopping power of the whole cluster turns out to be modified with respect to that of the same amount of uncorrelated charged debris. The correlated motion of heavy particles can be expected to occur also when dense conventional heavy-ion beams are used as drivers, provided their density be comparable, at least locally, to that of the cluster debris of the preceding example.

The effects of two-ion correlations on heavy-ion stopping power have already been extensively studied in the particular case of two collinear, equally charged ions moving in a classical plasma at a given velocity $\mathbf{v}_p$ [4]. The forces acting on the two-ion system and produced by the collective response of the plasma have also been investigated in Ref. [5]. The enhanced stopping power of close ions has been calculated both for fast ($v_p > v_{\rm the}$) and for slow ($v_p < v_{\rm the}$) projectiles [6]. The effects of a spatial average of the stopping power of two arbitrarily oriented ions have been evaluated by Arista [7].

In the present paper, the stopping power of a group of $N \gg 1$ ions, uniformly distributed in the physical space within a given volume and moving with almost equal velocities, is computed by applying averaging procedures over the configuration space.

Let us consider a distribution of N charged particles placed at $\mathbf{r}_j(t)$ ($j=1,\ldots,N$) in the laboratory frame. Let the particles move uniformly along straight trajectories, i.e., $\mathbf{r}_j(t)=\mathbf{r}_j+(t-t_0)\mathbf{v}_j$. The external charge distribution can then be written as

$$\rho_{\rm ext}(\mathbf{r},t)=\sum_{j=1}^{N} Z_{{\rm eff},j}e\,\delta(\mathbf{r}-(\mathbf{r}_j+\mathbf{v}_jt)), \tag{1}$$

where $t_0=0$ for simplicity. Due to the large mass of the projectiles with respect to the electrons, and limiting the analysis to suprathermal velocities (i.e., $v_j \gg v_{\rm the} \equiv \sqrt{T_e/m_e}$), we shall assume $\mathbf{v}_j=$const.

Let us introduce the position vector of the center of mass of the ensemble of N particles, $\mathbf{R}_0(t)\equiv\Sigma_{j=1}^{N}M_j(\mathbf{r}_j+\mathbf{v}_jt)/\Sigma_{j=1}^{N}M_j$, and its velocity $\mathbf{V}_0\equiv\Sigma_{j=1}^{N}M_j\mathbf{v}_j/\Sigma_{j=1}^{N}M_j$. Then, $\mathbf{R}_0(t)=\mathbf{R}_0+\mathbf{V}_0t$. By introducing the position of the jth ion and its velocity in the center-of-mass frame, i.e., $\Delta\mathbf{r}_j=\mathbf{r}_j-\mathbf{R}_0$ and $\Delta\mathbf{v}_j=\mathbf{v}_j-\mathbf{V}_0$, Eq. (1) can be written as

$$\rho_{\rm ext}(\mathbf{r},t)=\sum_{j=1}^{N} Z_{{\rm eff},j}e\,\delta(\mathbf{r}-(\mathbf{R}_0+\mathbf{V}_0t)-(\Delta\mathbf{r}_j+\Delta\mathbf{v}_jt)). \tag{2}$$

It is worth defining the following dimensionless variables:

$$\mathbf{r}\to\frac{\mathbf{r}}{\lambda_{\rm De}},\mathbf{v}\to\frac{\mathbf{v}}{v_{\rm the}},t\to t\omega_{\rm pe},\phi\to\frac{e\phi}{T_e},f_\alpha\to f_\alpha\frac{v_{\rm th\alpha}^3}{n_\alpha}. \tag{3}$$

Then the linearized Vlasov-Poisson system takes the form

$$\frac{\partial f_1}{\partial t}+\mathbf{v}\cdot\frac{\partial f_1}{\partial \mathbf{r}}+\frac{\partial \phi_1}{\partial \mathbf{r}}\cdot\frac{\partial f_0}{\partial \mathbf{v}}=0, \tag{4a}$$

$$\nabla^2\phi_1=-\sum_{j=1}^{N}Z_j\delta(\mathbf{r}-\mathbf{r}_j(t))+\int d^3v f_1(\mathbf{r},\mathbf{v},t), \tag{4b}$$

where $Z_j=Z_{{\rm eff},j}/N_D$, $N_D=n_0\lambda_{\rm De}^3\gg 1$, n_0 is the unperturbed electron density, and $Z_{{\rm eff},j}$ is the effective charge of the jth test ion inside the plasma, which we assume to be constant. $f_1(\mathbf{r},\mathbf{v},t)$ and $\phi_1(\mathbf{r},t)$ are the electron distribution function and the electrostatic potential at first order in the expansion in $Z_j \ll 1$. As a matter of fact, the linearized theory can be extended to $Z_j > 1$ provided that $Z_j/V_0^3 \ll 1$ [4,5]. Here, the interaction with plasma ions, the effects of Coulomb collisions on the stopping process, and the electromagnetic part of the interaction are not considered. $v_{\rm the}$, $\lambda_{\rm De}=\sqrt{T_e/4\pi n_0e^2}$, and $\omega_{\rm pe}=v_{\rm the}/\lambda_{\rm De}$ are the electron thermal velocity, Debye length, and plasma frequency, respectively.

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The stopping power of the system, i.e., the friction force acting in the direction of motion of the center of mass $\hat{\mathbf{e}}_0=\mathbf{V}_0/|\mathbf{V}_0|$, can then be found as

$$-\frac{dE}{dx}=\sum_{i=1}^{N} Z_i N_D \hat{\mathbf{e}}_0\cdot \left.\frac{\partial \phi_1}{\partial \mathbf{r}}\right|_{\mathbf{r}=\mathbf{r}_i(t)}$$

$$=-\frac{1}{(2\pi)^3}\sum_{i=1}^{N} Z_i^2 N_D \int d^3k\, \frac{\mathbf{k}\cdot\hat{\mathbf{e}}_0}{k^2}$$

$$\times \mathrm{Im}\left(\frac{1}{\varepsilon[\mathbf{k},\mathbf{k}\cdot(\mathbf{V}_0+\Delta\mathbf{v}_i)]}\right)-\frac{1}{(2\pi)^3}\sum_{i\neq j}^{1,\ldots,N} Z_i Z_j N_D$$

$$\times \int d^3k\, \frac{\mathbf{k}\cdot\hat{\mathbf{e}}_0}{k^2}\mathrm{Im}\left(\frac{\exp\{i\mathbf{k}\cdot[\mathbf{r}_{ij}+(\Delta\mathbf{v}_i+\Delta\mathbf{v}_j)t]\}}{\varepsilon[\mathbf{k},\mathbf{k}\cdot(\mathbf{V}_0+\Delta\mathbf{v}_i)]}\right), \tag{5}$$

where $\mathbf{r}_{ij}\equiv\mathbf{r}_i-\mathbf{r}_j$ and $\Delta\mathbf{v}_i=\mathbf{v}_i-\mathbf{V}_0$, at $t=0$. In the right-hand side (rhs) of Eq. (5), the first contribution represents the stopping power of N uncorrelated charged particles, while the second term contains the "interference" effect due to two-ion correlations. Moreover, $\varepsilon(\mathbf{k},\omega)=1+(1/k^2)W(\omega/k)$ is the longitudinal dielectric function and $W(\xi)=X(\xi)+iY(\xi)$ the plasma dispersion function [8], X and Y being its real and imaginary parts, respectively.

The aim of this paper is to compute the average value of Eq. (5) over given spatial and velocity distributions of the ensemble of the N particles. Let $f(\mathbf{r}_{ij})$ and $g(\Delta\mathbf{v}_i)$ be the distribution functions (both normalized to unity) of the interionic vectors $\mathbf{r}_{ij}$ and of the relative velocity $\Delta\mathbf{v}_i$ at the initial time $t=0$. Let us apply the averaging procedures:

$$\langle\cdot\rangle_{\mathbf{r}_{ij}}\equiv\int d^3r\cdot f(\mathbf{r}),\quad \langle\cdot\rangle_{\mathbf{v}_i}\equiv\int d^3v\cdot g(\mathbf{v}) \tag{6}$$

to Eq. (5). By assuming, for simplicity, that the distributions in Eq. (6) are isotropic both in the physical and in the velocity space, and that the charge states are the same for all the projectiles, i.e., $Z_j=Z$, for each j, then Eq. (5) can be put in the following form:

$$\left\langle\left\langle\left\langle-\frac{dE}{dx}\right\rangle_{\Delta\mathbf{r}_{ij}}\right\rangle_{\Delta\mathbf{v}_i}\right\rangle_{\Delta\mathbf{v}_j}=NS_u^{\mathrm{coll}}\{1+(N-1)\bar{\chi}^{\mathrm{coll}}\}$$

$$+NS_u^{\mathrm{sp}}\{1+(N-1)\bar{\chi}^{\mathrm{sp}}\}, \tag{7}$$

where the average value of the collective (single particle) contribution to the stopping power of a single uncorrelated ion,

$$S_u^{\mathrm{coll(sp)}}\equiv-\frac{Z^2N_D}{(2\pi)^3}\int_{k<1\,(k>1)} d^3k\, \frac{\mathbf{k}\cdot\hat{\mathbf{e}}_0}{k^2}$$

$$\times\mathrm{Im}\left\langle\frac{1}{\varepsilon(\mathbf{k},\mathbf{k}\cdot\mathbf{V}_0+\mathbf{k}\cdot\Delta\mathbf{v}_i)}\right\rangle_{\Delta\mathbf{v}_i}, \tag{8}$$

the average value of the collective (single particle) contribution to the stopping power of an ion due to mutual interference effects,

$$S_{\mathrm{int}}^{\mathrm{coll(sp)}}\equiv-\frac{Z^2N_D}{(2\pi)^3}\int_{k<1\,(k>1)} d^3k\, \frac{\mathbf{k}\cdot\hat{\mathbf{e}}_0}{k^2}\langle e^{i\mathbf{k}\cdot\mathbf{r}_{ij}}\rangle_{\mathbf{r}_{ij}}$$

$$\times\langle e^{-i\mathbf{k}\cdot\Delta\mathbf{v}_jt}\rangle_{\Delta\mathbf{v}_j}\mathrm{Im}\left\langle\frac{e^{i\mathbf{k}\cdot\Delta\mathbf{v}_it}}{\varepsilon(\mathbf{k},\mathbf{k}\cdot\mathbf{V}_0+\mathbf{k}\cdot\Delta\mathbf{v}_i)}\right\rangle_{\Delta\mathbf{v}_i}, \tag{9}$$

and the collective (single particle) component of the *effective vicinage function* [7]

$$\bar{\chi}^{\mathrm{coll(sp)}}\equiv\frac{S_{\mathrm{int}}^{\mathrm{coll(sp)}}}{S_u^{\mathrm{coll(sp)}}} \tag{10}$$

have been introduced. From inspection of Eqs. (7) and (10) it is argued that the following cases can occur. (i) If the N charged particles are sufficiently far away from each other, i.e., $|\mathbf{r}_{ij}|\gg1$, the phase factors in Eq. (9) give a negligible contribution when integrated over k. Then $S_{\mathrm{int}}\approx0$ and the rhs of Eq. (7) gives NS_u, that is the stopping power of N uncorrelated ions, each of charge state Z_{eff}. (ii) In the case of strong correlation, that is for $|\mathbf{r}_{ij}|\ll1$ and $|\Delta\mathbf{v}_j|=0$ (monochromatic ensemble of test ions), the phase factors are approximately unitary, $\bar{\chi}\approx1$, and the rhs of Eq. (7) approaches N^2S_u. Then the ensemble of N ions behaves almost as a single projectile of charge state NZ_{eff}. (iii) In intermediate situations $0<\bar{\chi}<1$ and even small values of $\bar{\chi}$ may have measurable effects provided N is sufficiently large.

Let us specify the distribution functions $f(\mathbf{r}_{ij})$ and $g(\Delta\mathbf{v}_i)$ relative to a simple physical system. Let the N equally charged ions be distributed uniformly in a spherical volume of radius $\Delta l/2$, that is,

$$f(\mathbf{r}_{ij})=\frac{3}{8\pi\Delta l^3}H(r_{ij})H(\Delta l-r_{ij}). \tag{11}$$

Moreover, let us assume that a small spread in projectile velocities, around the center of mass speed, be present, that is,

$$g(\Delta\mathbf{v}_j)=\frac{3}{4\pi\Delta v^3}H(\Delta v_j)H(\Delta v-\Delta v_j). \tag{12}$$

With these positions and by defining $\alpha_j\equiv k\Delta v_j$, $\mu_j\equiv\cos\vartheta_j$, $\vartheta_j\equiv\cos^{-1}(\mathbf{k}\cdot\Delta\mathbf{v}_j/k\Delta v_j)$, we can write

$$\mathrm{Im}\left\langle\frac{e^{-i\mathbf{k}\cdot\Delta\mathbf{v}_jt}}{\varepsilon(\mathbf{k},\mathbf{k}\cdot\mathbf{V}_0+\mathbf{k}\cdot\Delta\mathbf{v}_j)}\right\rangle_{\Delta\mathbf{v}_j}$$

$$=-\frac{3}{2(k\Delta v)^3}\int_0^{k\Delta v} d\alpha_j\alpha_j^2\int_{-1}^{+1}d\mu_j\frac{Y\cos(\alpha_j\mu_jt)}{(k^2+X^2)^2+Y^2}$$

$$-\frac{3}{2(k\Delta v)^3}\int_0^{k\Delta v} d\alpha_j\alpha_j^2\int_{-1}^{+1}d\mu_j\frac{(k^2+X^2)\sin(\alpha_j\mu_jt)}{(k^2+X^2)^2+Y^2}. \tag{13}$$

In Eq. (13) the first term in the rhs has a *resonant* nature in the high velocity limit; it describes the energy dissipation of

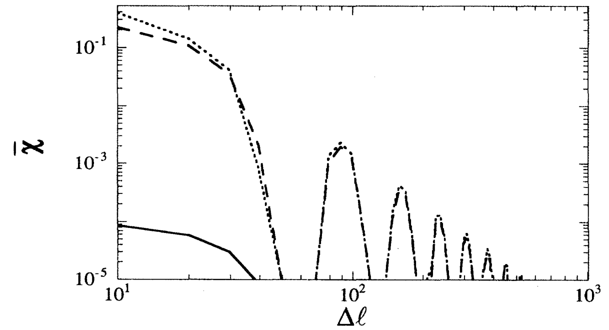

FIG. 1. The resonant part of the effective vicinage function $\bar{\chi}$ for $N=10^3$ charged particles is plotted versus Δl, for $t=10^2$, $V_0=10$, and $\Delta v=10^{-3}$ (dotted line), $t=10^4$ and $\Delta v=10^{-2}$ (full line), 10^{-3} (dashed line). The cold plasma approximation has been used. All quantities are in dimensionless units.

the projectiles in the interaction with resonant electrons [5]. The second term is *nonresonant* and describes internal forces which tend to rearrange the charge distribution. In Ref. [5] it was shown that, due to momentum conservation, this latter contribution is exactly zero for two particles moving with the same velocity. Here, it can be verified that this term is negligible for $\Delta v \ll 1$. Then, we shall consider only the *resonant* contributions to the two-ion correlations.

Moreover, since our aim is to investigate the stopping power of many charged particles in the presence of long-range correlations, which are due to the excitation of collective oscillations with wavelength larger than λ_{De}, it is possible to neglect the individual particle contributions to the k integrations in Eqs. (8) and (9) and to retain the collective contributions only. Then, we shall limit the relevant k integrals to values smaller than unity (*collective* approximation).

The effective vicinage function in Eq. (10) has been computed by retaining the first thermal corrections in the dielectric function (*warm plasma* approximation), i.e., by assuming $X(\xi) \approx -\xi^{-2} - 3\xi^{-4}$. In the *cold plasma* limit [that is, for $X(\xi) \approx -\xi^{-2}$, corresponding to large projectile velocities, $v_p \gg 1$], under strong correlation conditions, i.e., for $\Delta l < 1$ and $\Delta v \ll 1$, it is possible to obtain the approximate expression

$$\bar{\chi} \approx 1 - \frac{(\Delta l^2/20) + 3(\Delta v t)^2[\frac{1}{10} - (1/4V_0^2)]}{\ln V_0 - \frac{3}{10}(\Delta v^2/V_0^2)}. \quad (14)$$

In Fig. 1 the resonant part of the effective vicinage function of an ensemble of $N=1000$ particles is plotted as a function of the maximum spatial extension of the ion cluster Δl, for $\Delta v=10^{-3}$ and 10^{-2}, for $t=10^2$ and 10^4, and for $V_0=10$. It is observed that, besides the large correlation effect at small Δl values, small amplitude peaks appear even for large spatial extension of the cluster, which are reminiscent of the oscillating structure of the potential distribution behind each ion [9]. This effect at large Δl is expected to be reduced when distributions $f(\mathbf{r}_{ij})$ smoother than Eq. (12) are considered.

In Fig. 2 the resonant collective part of $\bar{\chi}$ is plotted versus V_0, for different spatial extension of an ensemble of $N=1000$ projectiles, $\Delta l=10,20,30,60,100$, and $t=100$. The warm plasma approximation has been used.

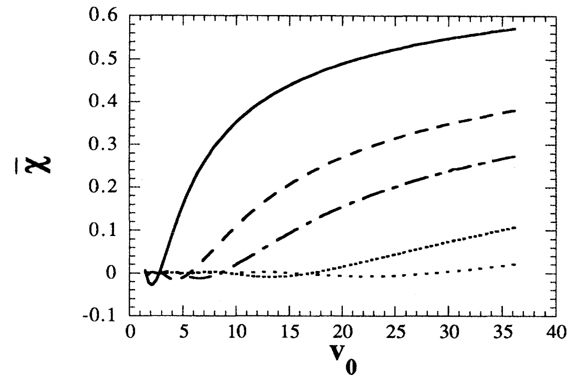

FIG. 2. The resonant part of $\bar{\chi}$ for $N=10^3$ projectiles is plotted versus V_0, for $\Delta l=10$ (full line), 20 (dashed line), 30 (dot-dashed line), 60 (densely dotted line), 100 (rarely dotted line), $\Delta v=10^{-3}$, and $t=10^2$. All quantities are in dimensionless units.

The presence of a velocity spread Δv makes the vicinage function, and then the stopping power of the whole system, depend explicitly on the time. It is possible with simple arguments to estimate the dependence of the *decorrelation time* on the system parameters. As has been demonstrated [4–7], the correlation between two fast ions of the ensemble occurs when one of the two particles, with velocity $\mathbf{v}_1$, falls inside the Cherenkov cone, with semiaperture $\varphi \approx \sqrt{3}/v_1 \ll 1$, excited behind the other charged particle [9]. We can estimate the "height" and a typical transverse dimension of the cone as $h \approx \pi v_1$ and $a \approx h\varphi$, respectively. The relative velocity between the two ions, of the order of Δv, makes them decorrelated after they have become separated by more than a linear spatial dimension of the Cherenkov cone, which can be taken as $h^{1/3}(h\varphi)^{2/3} \approx h\varphi^{2/3}$. The decorrelation time can then be estimated as

$$t_{\rm dec} \approx \frac{h\varphi^{2/3}}{\Delta v} \approx 3^{1/3}\pi \frac{V_0^{1/3}}{\Delta v}, \quad (15)$$

where $v_1 \approx V_0$ has been assumed.

Figure 3 shows $\bar{\chi}$ relative to $N=1000$ charged particles versus time, for $\Delta v=0.001$ and different values of Δl

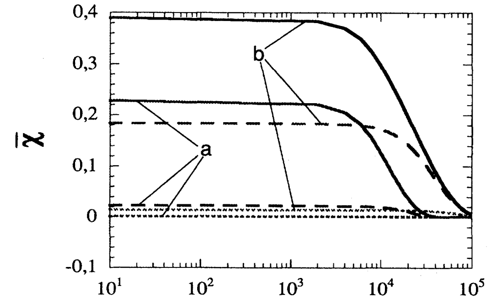

FIG. 3. The resonant part of $\bar{\chi}$ for $N=10^3$ particles is plotted versus t, for $V_0=11$ (a) and $V_0=36$ (b). Three Δl values are also considered: 10 (full lines), 30 (dashed lines), 100 (dotted lines). $\Delta v=10^{-3}$ and all quantities are in dimensionless units.

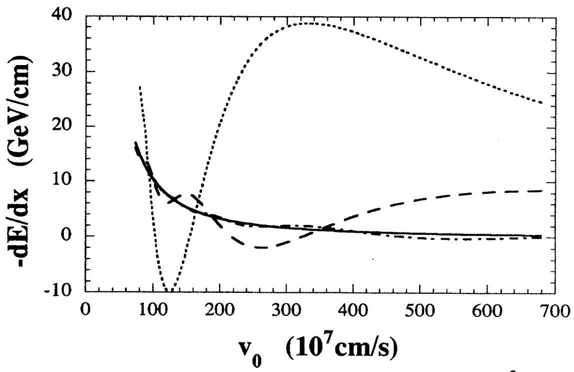

FIG. 4. The stopping power (in GeV/cm) of $N=10^3$ Pb ions, with $Z_{\rm eff}=50$, averaged over the distributions of Eqs. (11) and (12), is plotted as a function of V_0 (in 10^7 cm/s), for different values of Δl: 1 μm (dotted line), 2 μm (dashed line), 5 μm (dot-dashed line). The stopping power of 10^3 uncorrelated ions is also given for the sake of comparison (full line). The plasma parameters are $n_e=10^{18}$ cm^{-3}, $T_e=20$ eV (corresponding to $\lambda_{\rm De}\cong 3\times 10^{-2}$ μm and $Z\cong 1$). The picture corresponds to $t\cong 0.02$ ps.

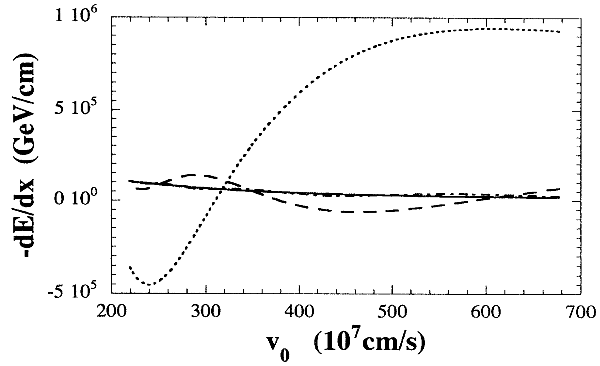

FIG. 5. The stopping power (in GeV/cm) of $N=10^3$ Bi ions, with $Z_{\rm eff}=80$, is plotted as a function of V_0 (in 10^7 cm/s), for different values of Δl: 10^2 Å (dotted line), 10^3 Å (dashed line), 5×10^3 Å (dot-dashed line). The full line refers to 10^3 uncorrelated Bi ions. A hydrogen ICF relevant plasma has been considered with $n_e=3\times 10^{22}$ cm^{-3} and $T_e=300$ eV (corresponding to $\lambda_{\rm De}\cong 7.4$ Å and $Z\cong 6$). The picture corresponds to $t=0.2$ fs.

(10,30,100), and of V_0 (11,36). The decorrelation after a time interval well approximated by Eq. (15), and slightly increasing with V_0, is found.

Finally we present the results of the computation of the stopping power of $N=1000$ charged particles in two typical configurations of physical interest: a Z-pinch plasma and an ICF relevant plasma. In Fig. 4 the stopping power as a function of the velocity V_0 of Pb ions with $Z_{\rm eff}=50$, moving in a Z-pinch H plasma with $n_e=10^{18}$ cm^{-3}, $T_e=20$ eV, is shown for $t\cong 0.02$ ps. Three values of Δl have been considered: 1 μm, 2 μm, 5 μm. The stopping power of 1000 uncorrelated ions is also plotted for the sake of comparison.

In Fig. 5 the stopping power (GeV/cm) versus V_0 (cm/s) of Bi ions with $Z_{\rm eff}=80$, injected in an ICF relevant H plasma with $n_e=3\times 10^{22}$ cm^{-3}, $T_3=300$ eV ($\lambda_{\rm De}\cong 7.4$ Å), at $t=0.2$ fs, is shown. Here the parameter Δl has the values 10^2, 10^3, and 5×10^3 Å. The full line corresponds to the case of 1000 uncorrelated ions.

We note that the basic assumption of constant particle velocity means that we have treated $dE/dx=0$ while calculating dE/dx itself. This hypothesis is justified whenever the decorrelation time turns out to be much less than the slowing-down time ($t_{\rm sd}$) of the charged projectiles. If we roughly estimate $t_{\rm dec}\approx 10^4\omega_{\rm pe}^{-1}$, and compute $t_{\rm sd}$ on the basis of the Coulomb collisional theory (e.g., see Ref. [10]), we get $t_{\rm dec}/t_{\rm sd}\approx 10^{-4}$, for $V_0=3\times 10^9$ cm/s, in the case of Fig. 4, and $t_{\rm dec}/t_{\rm sd}\approx 3\times 10^{-3}$, for $V_0=6\times 10^9$ cm/s, in the case of Fig. 5. These results agree with our assumption and show that the enhanced stopping power due to correlations affects the very initial part of the interaction. Therefore energy loss measurements of an ion ensemble (e.g., a cluster) through a plasma layer thinner than the corresponding range should be more appropriate for an experimental observation of the correlation effects.

In this paper the stopping power of an ensemble of a large number of projectiles in a warm plasma has been calculated by performing averages in the configuration space. The results of our analysis can be applied whenever high projectile concentrations are realized, at least locally. In particular, in the case of ICF plasmas the relevant beam densities are more likely to occur during the Coulomb explosion of molecular clusters in the plasma. In these cases an appreciable enhancement of the stopping power of the ensemble of ions is demonstrated, mostly during the initial part of the interaction. The existence of a velocity spread Δv, around the mean speed V_0, introduces a decorrelation time which turns out to be an increasing function of Δl, roughly proportional to Δv^{-1}, and almost independent of V_0.

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